A unified view of cross-world independence in multimediator causal models

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Abstract

Multi-mediator causal models provide a general framework for decomposing causal effects along specific mechanisms, but effect identification in the presence of treatment-induced confounding remains conceptually challenging. Path-specific effects (PSEs) provide one important formulation of such causal decomposition. In particular, existing approaches differ substantially in their counterfactual assumptions and in the interventions used to define effects through specific pathways. We use Single-World Intervention Graphs (SWIGs) as a unifying analytic device to formalize and compare three approaches to PSE identification: classical path-specific effects based on cross-world independence assumptions, interventional effects based on randomized mediator interventions, and separable effects based on decomposing the exposure into distinct causal components. Through this unified analysis, we show that these seemingly different approaches can, under appropriate conditions, lead to the same identification formulas despite relying on different causal assumptions and intervention semantics. The SWIG representation further reveals how each approach resolves the counterfactual conflicts created by treatment-induced confounding. Our results highlight a fundamental distinction between statistical identification and causal interpretation: the same observed-data functional can represent different causal questions depending on how the underlying counterfactual worlds are constructed. This unified perspective clarifies relationships among major approaches in multimediator settings and guides alignment of the estimand, identification assumptions, and scientific mechanism of interest.